

# Effects of Large CP Phases on the Proton Lifetime in Supersymmetric Unification

Tarek Ibrahim<sup>a,b</sup> and Pran Nath<sup>b</sup>

a. Department of Physics, Faculty of Science, University of Alexandria, Alexandria, Egypt

b. Department of Physics, Northeastern University, Boston, MA 02115-5000, USA

## Abstract

The effects of large CP violating phases arising from the soft SUSY breaking parameters on the proton lifetime are investigated in supersymmetric grand unified models. It is found that the CP violating phases can reduce as well as enhance the proton lifetime depending on the part of the parameter space one is in. Modifications of the proton lifetime by as much as a factor of 2 due to the effects of the CP violating phases are seen. The largest effects arise for the lightest sparticle spectrum in the dressing loop integrals and the effects decrease with the increasing scale of the sparticle masses. An analysis of the uncertainties in the determination of the proton life time due to uncertainties in the quark masses and in the other input data is also given. These results are of import in the precision predictions of the proton lifetime in supersymmetric unification both in GUT and in string models when the soft SUSY breaking parameters are complex.

# 1 Introduction

It is well known that there are new sources of CP violation in supersymmetric theories which arise from the soft SUSY breaking sector of the theory. The normal size of such phases is  $O(1)$  and an order of magnitude estimate shows that such large phases would lead to a conflict with the current experimental limits on the electron[1] and on the neutron electric dipole moment[2]. The conventional ways suggested to avoid this conflict is either to assume that the phases are small[3, 4] or that the SUSY spectrum is heavy[5]. However, recently it was demonstrated[6] that this need not to be the case and indeed there could be consistency with experiment even with large CP violating phases and a light spectrum due to an internal cancellation mechanism among the various contributions to the EDMs. The above possibility has led to a considerable further activity [7, 8, 9] and the effects of large CP phases under the cancellation mechanism have been investigated in dark matter with the EDM constraints[10], in  $g_\mu - 2$ [11] and in other low energy physics phenomena[12].

In this paper we investigate the effects of large CP violating phases on nucleon stability in supersymmetric grand unification with baryon and lepton number violating dimension five operators[13, 14, 15]. The main result of this analysis is that the dressing loop integrals that enter in the supersymmetric proton decay analysis are modified due to the effect of the large CP violating phases. The CP effects on the proton life time are most easily exhibited by considering  $R_\tau$  defined in Eq.(22) which is the ratio of the p lifetime with phases and without phases.  $R_\tau$  is largely independent of the GUT structure which cancels out in the ratio. Since the dressing loop integrals enter in the proton decay lifetime in both GUT and string models which contain the baryon and the lepton number violating dimension five operators, the phenomena of CP violating effects on the proton lifetime should hold for a wide range of models both of GUT and of string variety[16]. However, for concreteness we will consider first the simplest SU(5) supersymmetric grand unified model, and then consider a non-minimal extension. As discussed above similar analyses should hold for a wider class of models and so what we do below should serve as an illustration of the general idea of the effect of large CP phases on the proton lifetime.

The outline of the paper is as follows: In Sec.2 we give a theoretical analysis of the effects of CP violating phases on proton decay in the minimal supersymmetric SU(5) model for specificity. In Sec.3 we discuss the numerical effects of the CP

violating phases on  $R_\tau$  under the EDM constraints. A non-minimal extension is also discussed and an analysis of the uncertainties in the predictions of the proton life time due to uncertainties in the quark masses, in  $\beta_p$  and in the KM matrix elements is given. Conclusions are given in Sec.4.

## 2 Theoretical analysis of CP violating phases on proton decay in supersymmetric GUTs

In the minimal supergravity unified model (mSUGRA)[17] the soft SUSY breaking can be parameterized by  $m_0$ ,  $m_{\frac{1}{2}}$ ,  $A_0$ , and  $\tan \beta$ , where  $m_0$  is the universal scalar mass,  $m_{\frac{1}{2}}$  is the universal gaugino mass,  $A_0$  is the universal trilinear coupling all taken at the GUT scale, and  $\tan \beta = \langle H_2 \rangle / \langle H_1 \rangle$  is the ratio of the Higgs VEVs where  $H_2$  gives mass to the up quark and  $H_1$  gives mass to the down quark and the lepton. In addition, the effective theory below the GUT scale contains the Higgs mixing parameter  $\mu$  which enters in the superpotential in the term  $\mu H_1 H_2$ . In the presence of CP violation one finds that the minimal model contains two independent CP violating phases which can be taken to be  $\theta_\mu$ , which is the phase of  $\mu$  and  $\alpha_{A_0}$  which is the phase of  $A_0$ .

For more general situations when one allows for non-universalities, the soft SUSY breaking sector of the theory brings in more CP violating phases. Thus unlike the case of mSUGRA here the  $U(1) \times SU(2) \times SU(3)$  gaugino masses  $\tilde{m}_i$  ( $i=1,2,3$ ) can have arbitrary phases, i.e.,

$$\tilde{m}_i = |\tilde{m}_i| e^{i\xi_i} \quad (i = 1, 2, 3) \quad (1)$$

While in the universal case a field redefinition can eliminate the common phase of the gaugino masses, here one finds that the difference of the gaugino phases does persist in the low energy theory and in fact is found to be a useful tool in arranging for the cancellation mechanism to work for the satisfaction of the EDMs. In the following analysis we carry out an analysis of the proton decay with the most general allowed set of CP violating phases. The definition of the mass matrices for charginos, neutralinos and for squarks and sleptons have been explicitly exhibited in Ref.[18] and we refer the reader to this paper for details. The focus of the present work is to analyze the effects of CP violating phases on p decay and to estimate its size. For the sake of concreteness we begin with a discussion of the simplest grand unification model, i.e., the minimal SU(5) model. However, the technique

discussed here to include the CP effects on p decay can be used to analyse the CP violating effects for any supersymmetric unified model with baryon and lepton number violating dimension five operators. This class includes string models.

As mentioned above we consider for concreteness and simplicity the minimal SU(5) model whose matter interactions are given by [13, 14, 15]

$$W_Y = -\frac{1}{8}f_{1ij}\epsilon_{uvwxy}H_1^u M_i^{vw} M_j^{xy} + f_{2ij}\bar{H}_{2u}\bar{M}_{iv}M_j^{uv} \quad (2)$$

where  $M_u, M^{uv}$  are the  $\bar{5}, 10$  plet representations of SU(5), and  $H_1, H_2$  are the  $\bar{5}, 5$  of SU(5). After the breakdown of the GUT symmetry and integration over the Higgs triplet fields the effective dimension five interactions below the GUT scale which governs p decay is given by [13, 14, 15]

$$\mathcal{L}_{5L} = \frac{1}{M}\epsilon_{abc}(Pf_1^u V)_{ij}(f_2^d)_{kl}(\tilde{u}_{Lbi}\tilde{d}_{Lcj}(\bar{e}_{Lk}^c(Vu_L)_{al} - \bar{\nu}_k^c d_{Lal}) + \dots) + H.c. \quad (3)$$

$$\mathcal{L}_{5R} = -\frac{1}{M}\epsilon_{abc}(V^\dagger f^u)_{ij}(PVf^d)_{kl}(\bar{e}_{Ri}^c u_{Raj}\tilde{u}_{Rck}\tilde{d}_{Rbl} + \dots) + H.c. \quad (4)$$

where  $\mathcal{L}_{5L}$  and  $\mathcal{L}_{5R}$  are the LLLL and RRRR lepton and baryon number violating dimension 5 operators, V is the CKM matrix and  $f_i$  are related to quark masses, and  $P_i$  appearing in Eqs.(3) and (4) are the generational phases given by  $P_i = (e^{i\gamma_i})$ ,  $\sum_i \gamma_i = 0$  (i=1,2,3).

The baryon and the lepton number violating dimension five operators must be dressed by the chargino, the gluino and the neutralino exchanges to generate effective baryon and lepton number violating dimension six operators at low energy (Some examples of dressing loop diagrams are given in Fig.1). It is in this process of dressing of the dimension five operators that the CP violating phases of the soft SUSY breaking sector enter in the proton decay amplitude. The CP phases enter the dressings in two ways, via the mass matrices of the charginos, the neutralinos and the sfermions, and via the interaction vertices. Taking account of this additional complexity, the analysis for computing the proton decay amplitudes follows the usual procedure. Thus to dress the dimension five operators the squark and slepton fields must be eliminated in terms of their sources. As an example, the up squarks in the presence of CP violating phases can be eliminated using the relations

$$\begin{aligned} \tilde{u}_{iL} &= 2 \int [\Delta_{ui}^L L_{ui} + \Delta_i^{LR} R_{ui}] \\ \tilde{u}_{iR} &= 2 \int [\Delta_{ui}^R R_{ui} + \Delta_i^{RL} L_{ui}] \end{aligned} \quad (5)$$

where

$$L_{ui} = \frac{\delta L_I}{\delta \tilde{u}_{iL}^\dagger}, R_{ui} = \frac{\delta L_I}{\delta \tilde{u}_{iR}^\dagger} \quad (6)$$

Here  $L_I$  is the sum of fermion-sfermion-gluino, fermion-sfermion-chargino and fermion-sfermion-neutralino interactions and

$$\begin{aligned} \Delta_{ui}^L &= [|D_{ui11}|^2 \Delta_{ui1} + |D_{ui12}|^2 \Delta_{ui2}] \\ \Delta_{ui}^R &= [|D_{ui21}|^2 \Delta_{ui1} + |D_{ui22}|^2 \Delta_{ui2}] \end{aligned} \quad (7)$$

and

$$\begin{aligned} \Delta_{ui}^{LR} &= -D_{ui11} D_{ui12} [\Delta_{ui1} - \Delta_{ui2}] \\ \Delta_{ui}^{RL} &= -D_{ui11} D_{ui12}^* [\Delta_{ui1} - \Delta_{ui2}]. \end{aligned} \quad (8)$$

Here  $\tilde{u}_{i1}$  and  $\tilde{u}_{i2}$  are the squark mass eigenstates for the squark flavors  $u_{i1}$  and  $u_{i2}$  and  $\Delta_{u_{i1}}$  and  $\Delta_{u_{i2}}$  are the corresponding propagators, and  $D_{ui}$  is the diagonalizing matrix for the  $\tilde{u}_i$  squarks, i.e.,

$$D_{ui}^\dagger M_{\tilde{u}_i}^2 D_{ui} = \text{diag}(M_{\tilde{u}_{i1}}^2, M_{\tilde{u}_{i2}}^2) \quad (9)$$

We note the special arrangement of the complex quantities and their complex conjugates in Eqs.7 and 8. Specifically we note that while in the absence of CP phases  $\Delta_{ui}^{LR} = \Delta_{ui}^{RL}$  this is not the case in the presence of CP phases and in general one has  $\Delta_{ui}^{LR} \neq \Delta_{ui}^{RL}$  as is seen from Eqs.(8).  $L_{ui}$  and  $R_{ui}$  defined by Eq.(6) receive contributions from the chargino, the neutralino and the gluino exchanges.

Following the standard procedure[13, 14, 15] one obtains the effective dimension six operators for the baryon and the lepton number violating interaction arising from dressing of the dimension five operators. From this effective interaction one obtains the proton lifetime decay widths for various modes using the effective Lagrangian methods. We limit ourselves here to the dominant decay mode  $p \rightarrow \bar{\nu}_i K^+$ . Including the CP violating effects the decay width for this process is given by

$$\begin{aligned} \Gamma(p \rightarrow \bar{\nu}_i K^+) &= \frac{\beta_p^2 m_N}{M_{H_3}^2 32\pi f_\pi^2} \left(1 - \frac{m_K^2}{m_N^2}\right)^2 |\mathcal{A}_{\nu_i K}|^2 A_L^2 (A_S^L)^2 \\ &\quad \left| \left(1 + \frac{m_N(D + 3F)}{3m_B}\right) (1 + \mathcal{Y}_i^{tk} + (e^{-i\xi_3} \mathcal{Y}_{\tilde{g}} + \mathcal{Y}_{\tilde{Z}}) \delta_{i2} + \frac{A_S^R}{A_S^L} \mathcal{Y}_1^R \delta_{i3}) \right. \\ &\quad \left. + \frac{2}{3} \frac{m_N}{m_B} D (1 + \mathcal{Y}_3^{tk} - (e^{-i\xi_3} \mathcal{Y}_{\tilde{g}} - \mathcal{Y}_{\tilde{Z}}) \delta_{i2} + \frac{A_S^R}{A_S^L} \mathcal{Y}_2^R \delta_{i3}) \right|^2 \end{aligned} \quad (10)$$

where

$$\mathcal{A}_{\nu_i K} = (\sin 2\beta M_W^2)^{-1} \alpha_2^2 P_2 m_c m_i^d V_{i1}^\dagger V_{21} V_{22} [\mathcal{F}(\tilde{c}; \tilde{d}_i; \tilde{W}) + \mathcal{F}(\tilde{c}; \tilde{e}_i; \tilde{W})] \quad (11)$$

In the above  $A_L(A_S)$  are the long (short) suppression factors, D,F,  $f_\pi$  are the effective Lagrangian parameters, and  $\beta_p$  is defined by  $\beta_p U_L^\gamma = \epsilon_{abc} \epsilon_{\alpha\beta} < 0 | d_{aL}^\alpha u_{bL}^\beta u_{cL}^\gamma | p >$  where  $U_L^\gamma$  is the proton wavefunction. Theoretical determinations of  $\beta_p$  lie in the range  $0.003 - 0.03 \text{ GeV}^3$ . Perhaps the more reliable estimate is from lattice gauge calculations which gives[19]  $\beta_p = (5.6 \pm 0.5) \times 10^{-3} \text{ GeV}^3$ .

Aside from the explicit CP phases via the exponential factor  $e^{-i\xi_3}$  in Eq.(10), CP effects enter dominantly in  $\mathcal{F}$ 's which are the dressing loop integrals. For the chargino exchange in the presence of CP violating phases one has

$$\begin{aligned} \mathcal{F}(\tilde{u}_i; \tilde{d}_j; \tilde{W}) = -32\pi^2 i \int \sum_{A=1,2} [\Delta_{uai}^L S_{A1}^* - \Delta_{uai}^{LR} \epsilon_i^u S_{A2}^*] \\ \tilde{G}_A [\Delta_{dj}^L U_{A1}^* - \Delta_{dj}^{LR} \epsilon_j^d U_{A2}^*] \end{aligned} \quad (12)$$

Here  $\tilde{G}_A$  (A=1,2) are the propagators for the chargino mass eigenstates and the matrices U and S enter in the biunitary transformations to diagonalize the chargino mass matrix  $M_C$  such that

$$U^* M_C S^{-1} = \text{diag}(\tilde{m}_{\chi_1^+}, \tilde{m}_{\chi_2^+}) \quad (13)$$

In Eq.(10) the quantities  $\mathcal{Y}_i^{tk}$  are the corrections due to the chargino exchanges involving third generation squarks,  $\mathcal{Y}_{\tilde{g}}$  is the contribution from the gluino exchange,  $\mathcal{Y}_{\tilde{Z}}$  is the contribution from the neutralino exchange, and  $\mathcal{Y}_i^R$  are the contributions from the dressing of the RRRR dimension 5 operators.

The gluino exchange contribution  $\mathcal{Y}_{\tilde{g}}$  is given by

$$\mathcal{Y}_{\tilde{g}} = \frac{4}{3} \frac{P_1}{P_2} \frac{\alpha_3}{\alpha_2} \frac{m_u V_{11}}{m_c V_{21} V_{21}^\dagger V_{22}} \frac{\mathcal{H}(\tilde{u}; \tilde{d}; \tilde{g}) - \mathcal{H}(\tilde{d}; \tilde{d}; \tilde{g})}{\mathcal{F}(\tilde{c}; \tilde{s}; \tilde{W}) + \mathcal{F}(\tilde{c}; \tilde{\mu}; \tilde{W})} \quad (14)$$

The function  $\mathcal{H}$  is defined by  $\mathcal{H}(\tilde{u}; \tilde{d}; \tilde{g}) = f(\tilde{m}_u; \tilde{m}_d; \tilde{m}_g)$  where f is defined by Eq.(19) below. The contributions  $\mathcal{Y}_i^R$  from the dressing of the RRRR dimension five operators are given by

$$\mathcal{Y}_1^R = \frac{P_1}{P_2} \frac{m_t m_d V_{11} V_{32} V_{33}^\dagger}{m_c m_b V_{21} V_{22} V_{31}^\dagger} \frac{\mathcal{Q}(\tilde{\tau}; \tilde{t}; \tilde{W})}{\mathcal{F}(\tilde{c}; \tilde{b}; \tilde{W}) + \mathcal{F}(\tilde{c}; \tilde{\tau}; \tilde{W})} \quad (15)$$

and by

$$\mathcal{Y}_2^R = \frac{P_1}{P_2} \frac{m_t m_s V_{31} V_{12} V_{33}^\dagger}{m_c m_b V_{21} V_{22} V_{31}^\dagger} \frac{\mathcal{Q}(\tilde{\tau}; \tilde{t}; \tilde{W})}{\mathcal{F}(\tilde{c}; \tilde{b}; \tilde{W}) + \mathcal{F}(\tilde{c}; \tilde{\tau}; \tilde{W})} \quad (16)$$

where  $\mathcal{Q}$ 's are defined as follows:

$$\mathcal{Q}(\tilde{\tau}; \tilde{t}; \tilde{W}) = -32\pi^2 i \left( \frac{m_\tau}{\sqrt{2}M_W \cos \beta} \right) \int \sum_{A=1,2} \Delta_\tau^R U_{A2}^* [\epsilon_t \Delta_t^R S_{A2}^* - \Delta_t^{RL} S_{A1}^*] \tilde{G}_A \quad (17)$$

The dressing loop integrals can be expressed in terms of the basic integral

$$f(\mu_i, \mu_j, \mu_k) = -16\pi^2 i \int \Delta_i \Delta_j \tilde{G}_k \quad (18)$$

where

$$f(\mu_i, \mu_j, \mu_k) = \frac{\mu_k}{\mu_j^2 - \mu_k^2} \left[ \frac{\mu_j^2}{\mu_i^2 - \mu_j^2} \ln \frac{\mu_i^2}{\mu_j^2} - \frac{\mu_k^2}{\mu_i^2 - \mu_k^2} \ln \frac{\mu_i^2}{\mu_k^2} \right] \quad (19)$$

In the limit of no CP violation the analysis limits correctly to the previous results which do not include CP violating effects.

### 3 Numerical effects of CP violating phases on proton decay

We discuss now the numerical size of the effects of CP phases on the proton lifetime. As is obvious from our discussion above the proton lifetime is highly model dependent. Specifically there are two main factors that govern the lifetime of the proton. One of these depends on the nature of the GUT sector, i.e., if the GUT group is SU(5), SO(10),  $E_6$ ,...etc and on the nature of the GUT interaction, e.g., on the GUT Higgs structure, while the second factor that controls proton decay is the sparticle spectrum and the sparticle interactions that enter in the dressing loop integrals.

In the simplest SU(5) GUT model with two Higgs multiplets  $H_1$  and  $H_2$ , GUT physics enters mainly via the Higgs triplet mass  $M_{H_3}$ . The proton decay lifetime is significantly affected if we were to change the GUT structure. Thus, for example, if one had many Higgs triplets[20],  $H_i$  and  $\bar{H}_i$  ( $i=1,...,n$ ) where only the triplets  $H_1$  and  $\bar{H}_1$  couple with matter, i.e.,

$$W_3^{triplet} = \bar{H}_1 J + \bar{J} H_1 + \bar{H}_i M_{ij} H_j \quad (20)$$

then the effective interaction on eliminating the Higgs triplets is

$$W_4^{eff} = -\bar{J} (M_{11}^{-1}) J \quad (21)$$

Here one finds that the effective Higgs triplet mass is  $M_{H_3}^{eff} = ((M^{-1})_{11})^{-1}$ . The model with  $M_{H_3}$  and  $M_{H_3}^{eff}$  would have very similar CP violating effects for the

same low energy sparticle spectrum. The reason is that the CP effects are largely governed by the nature of the low energy physics, e.g., the sparticle mass spectrum and the couplings of the sparticles with matter. Thus we expect similar size CP violating effects in models with different GUT structures but with similar size sparticle spectrum

To discuss the CP violating effects on the proton lifetime it is useful to consider the ratio  $R_\tau(p \rightarrow \bar{\nu} + K^+)$  defined by

$$R_\tau(p \rightarrow \bar{\nu} + K^+) = \tau(p \rightarrow \bar{\nu} + K^+)/\tau_0(p \rightarrow \bar{\nu} + K^+) \quad (22)$$

Here  $\tau(p \rightarrow \bar{\nu} + K^+)$  is the proton lifetime with CP violating phases and  $\tau_0(p \rightarrow \bar{\nu} + K^+)$  is the lifetime without CP phases. This ratio is largely model independent. Thus most of the model dependent features such as the nature of the GUT or the string model would be contained mostly in the front factors such as the Higgs triplet mass, the quark masses, the  $A_S$  and  $A_L$  suppression factors all of which cancel out in the ratio. Similarly the quantity  $\beta_p$  which is poorly known cancels out in the ratio as do the KM matrix elements.

We analyze  $R_\tau$  under the constraints that CP violating phases obey the experimental limits on the electron and the neutron EDMs. For the electron and for the neutron the current experimental limits are[1, 2]

$$|d_e| < 4.3 \times 10^{-27} ecm, \quad |d_n| < 6.3 \times 10^{-26} ecm \quad (23)$$

We are interested in the effects of large phases on the proton lifetime and for these to satisfy the EDM constraints we use the cancellation mechanism. In Fig.2 we present five cases where for different inputs the electron EDM is plotted as a function of  $\theta_\mu$ . An analysis of the neutron EDM for the same input is given in Fig.3. One finds the cancellation mechanism produces several regions where the EDM constraints are satisfied. In Fig.4 we give a plot of  $R_\tau$  for the same set of inputs as in Figs. 2 and 3. The analysis shows that  $R_\tau$  is a sensitive function of the CP phase  $\theta_\mu$  and variations of a factor of around 2 can occur. We also note that both a suppression as well as an enhancement of the proton lifetime can occur as a consequence of the CP violating effects. Interestingly the largest CP effects on  $R_\tau$  occur here at the points of maximum cancellation in the EDMs as may be seen by a comparison of Figs.2, 3 and 4. The variations in  $R_\tau$  due to the phases arise because of constructive and destructive interference between the exchange contributions of chargino 1 ( $\chi_1^+$ ) and chargino 2 ( $\chi_2^+$ ) (see Fig.1). We give an illustration of this



phenomenon in Table 1. The analysis of Table 1 exhibits the cancellation in the imaginary part of the amplitude for the decay process  $p \rightarrow \bar{\nu} + K^+$  from chargino 1 and chargino 2, and this cancellation leads to an enhancement in the p lifetime ratio for this case.

It is possible to promote each of the cancellation points in Figs.2 and 3 into a trajectory in the  $m_0 - m_{\frac{1}{2}}$  plane by scaling upwards by a common scale transformation[8]

$$m_0 \rightarrow \lambda m_0, \quad m_{\frac{1}{2}} \rightarrow \lambda m_{\frac{1}{2}} \quad (24)$$

The size of the sparticle spectrum depends on the scale  $\lambda$ . In general, the larger the value of  $\lambda$  the heavier is the sparticle spectrum and correspondingly smaller is the CP effect on the dressing loop as demonstrated in Fig.5. For the minimal SU(5) case one needs a relatively heavy spectrum with some of the sparticle masses  $\sim 1$  TeV to stabilize the proton which has the current experimental limit for the  $p \rightarrow \bar{\nu} K$  decay mode of  $\tau(p \rightarrow \bar{\nu} K) > 5.5 \times 10^{32}$  yr[21]. Because of the heaviness of the sparticle spectrum, the CP effects for the minimal SU(5) model are typically small, i.e., of order only a few percent. Larger CP effects can occur in non-minimal models where one has several Higgs triplets. Thus we consider an example where one has two pairs of heavy Higgs triplets with the Higgs triplet mass matrix given by

$$\begin{pmatrix} 0 & a\Lambda \\ a\Lambda & M_2 \end{pmatrix} \quad (25)$$

Such a structure can arise, for example, in an SO(10) model[22] with two **10's** of Higgs and a **45** of Higgs with a superpotential of the type  $W_H = M_2 \mathbf{10}_{2H}^2 + \Lambda \mathbf{10}_{1H} \mathbf{45}_H \mathbf{10}_{2H}$ . After the **45** of Higgs develops a VEV  $\langle \mathbf{45}_H \rangle = (a, a, a, 0, 0) \times i\sigma_2$  one finds that only one pair of Higgs doublets remain massless while the Higgs triplets  $(\bar{H}_{t1}, \bar{H}_{t2})$  and  $(H_{t1}, H_{t2})$  have the mass matrix given by Eq.(25). In this case one has  $M_{eff} = a^2 \Lambda^2 / M_2$  and one can arrange for proton stability even with a light spectrum by an adjustment of the parameters  $a\Lambda$  and  $M_2$ .

Finally we discuss the current uncertainties in the proton lifetime predictions. Uncertainties arise from the errors in the quark masses, in  $\beta_p$  and in the KM matrix elements. The largest source of uncertainties arises from the strange quark mass ( $m_s$ ). There are several determinations of  $m_s$ :  $m_s = 193 \pm 59$  MeV[23],  $m_s = 200 \pm 70$  MeV[24],  $m_s = 170 \pm 50$  MeV[25],  $m_s = 155 \pm 15$  MeV[26], all evaluated at the scale 1GeV. We take for our average  $m_s = 180 \pm 50$  MeV. For the charm quark mass ( $m_c$ ) we use  $m_c = 1.4 \pm 0.2$  GeV[27] while for the bottom quark

mass ( $m_b$ ) we use  $m_b = 4.74 \pm 0.14$  GeV[27]. The contributions from the first generation quarks are small and are not the sources of any significant uncertainty in the p lifetime. The errors in the KM matrix elements are of a subleading order for the  $\bar{\nu}K$  mode but are still significant enough to be included. We use the results of Ref.[27] for the allowed ranges of the KM matrix elements. For  $\beta_p$  we use the result of the lattice gauge analysis of Ref.[19]. In Fig.6 we exhibit the error corridor for the proton lifetime for the case (1) of Fig.2 with  $M_2/a\Lambda = 0.01, M_2 = M_G$ . One finds that given the current errors in the input data the predictions for the proton lifetime has an uncertainty of about a factor of 2 ( $1_{-0.5}^{1.5}$ ) on either side of the mean. A similar analysis holds for the cases(2-5) of Fig.2. We note that the uncertainties in the predictions of the proton lifetime is of the same order as the size of the CP violating effects. It is for this reason that we choose to exhibit the results of our analysis in Figs. 4 and 5 in terms of the ratio  $R_\tau$  since the effects of the uncertainties cancel in the ratio. The analysis also shows that an improvement in the determination of the quark masses and of  $\beta_p$  is essential for a more precise prediction of the proton lifetime in supersymmetric unification of the type discussed here. The reduction of the error in the prediction of p lifetime will also help to define the CP effects on proton decay when such a decay is experimentally observed.

In summary the CP violating effects on the proton lifetime are relatively large if the sparticle spectrum entering the dressing loop integrals is relatively light and the CP violating effects get progressively smaller as the scale of the sparticle spectrum entering the dressing loops gets progressively larger. The current experimental limits on the sparticle masses allow for a relatively light sparticle spectrum, i.e., significantly smaller than 1 TeV. This means that there exists the possibility of significant CP violating effects on the proton lifetime. However, the minimal SU(5) model does not support the scenario with a light spectrum and thus the CP violating effects for the case of the minimal model are small. However, for the non-minimal case proton stability can occur even for a relatively light spectrum due to suppression from a more complicated Higgs triplet sector. In these types of models CP violating effects can be significant.

## 4 Conclusion

In this paper we have investigated the effects of CP violating phases arising from the soft SUSY breaking sector of the theory on the proton decay amplitudes. It is

found that the CP effects can increase or decrease the proton decay rates and that the size of their effect depends sensitively on the region of the parameter space one is in. Effects as large as a factor of 2 are seen to arise from CP violating phases in the part of the parameter space investigated and even larger effects in the full parameter space may occur. It is found that the CP violating effects in the minimal SU(5) model are typically small since a relatively heavy sparticle spectrum is needed to stabilize the proton in this case and a heavy spectrum suppresses the CP effects in the dressing loop integral. However, significantly larger CP effects on the  $p$  lifetime are possible in non-minimal models with more than one pair of Higgs triplets since in these models the proton can be stabilized with a relatively light sparticle spectrum. We also investigated the uncertainties in the  $p$  lifetime predictions due to uncertainties in the quark masses, in  $\beta_p$  and in the KM matrix elements. We find that these uncertainties modify the proton lifetime by a factor of 2 around the mean value. The observations arrived at in this analysis would be applicable to a wide class of models, including GUT models and string models with dimension five baryon and lepton number violating operators

Note Added: After the paper was submitted for publication an improved limit on  $p \rightarrow \bar{\nu}_\mu K^+$  mode of  $\tau(p \rightarrow \bar{\nu}_\mu K^+) > 1.9 \times 10^{33}$  yr has been reported[28]. The new limit does not affect the conclusions arrived at in this paper.

### Acknowledgements

This research was supported in part by NSF grant PHY-9901057

Table 1:CP effects on Chargino dressings.

case	Chargino 1	Chargino 2
$(\text{Re}A_{\nu_\mu}^{W_i}/ A_{\nu_{\mu 0}} , \text{Im}A_{\nu_\mu}^{W_i}/ A_{\nu_{\mu 0}} )$	$(-.22, -.89)$	$(-.026, .2)$
case	$(\text{Re}A_{\nu_\mu}/ A_{\nu_{\mu 0}} , \text{Im}A_{\nu_\mu}/ A_{\nu_{\mu 0}} )$	$ A_{\nu_\mu} / A_{\nu_{\mu 0}} $
sum 1&2	$(-.25, -.69)$	.74

Table caption: The table gives an analysis of the dressing loop integrals for dressings with Charginos 1&2 (see Fig.1) and their sum for the case when  $m_0 = 71$  GeV,  $m_{\frac{1}{2}} = 148$  GeV,  $\tan \beta = 2$ ,  $\theta_\mu = 1.4$ ,  $\xi_1 = 0.3$ ,  $\xi_2 = 1.8$ ,  $\xi_3 = 0$ , where all phases are in radians. The analysis shows cancellations in the dressings between Chargino 1 and Chargino 2 for the case with phases.

### Figure Captions

Fig.1: Examples of the dressing of LLLL baryon-number-violating dimension five

operators by chargino, gluino and neutralino exchanges that contribute to the proton decay. Cancellation among diagrams such as between  $\chi_1^+$  and  $\chi_2^+$  exchanges can lead to an enhancement of the proton lifetime. The dressings of the RRRR dimension five operators is also exhibited.

Fig.2: Plot of  $\text{Log}_{10}|d_e|$  vs  $\theta_\mu$  exhibiting cancellations where the five curves correspond to the five sets of input for the parameters  $\tan\beta$ ,  $m_0$ ,  $m_{1/2}$ ,  $\xi_1$ ,  $\xi_2$ ,  $\xi_3$ ,  $\alpha_{A_0}$ , and  $A_0$  given by (1)2,71,148,  $-1.15, -1.4, 1.27, -0.4, 4$  (dotted), (2)2,71,148,  $-0.87, -1.0, 1.78, -0.4, 4$  (solid), (3)4,550,88,  $.5, -1.55, 1.5, .6, .8$  (dashed), (4)4,750,88,  $1.5, 1.6, 1.7, .6, .8$  (long dashed), and (5)2,71,148,  $.55, 1., 1.35, -0.4, 4$  (dot-dashed). All masses are in GeV and all phases are in radians.

Fig.3: Plot of  $\text{Log}_{10}|d_n|$  vs  $\theta_\mu$  exhibiting cancellations where the five curves correspond to the five sets of input for the parameters  $\tan\beta$ ,  $m_0$ ,  $m_{1/2}$ ,  $\xi_1$ ,  $\xi_2$ ,  $\xi_3$ ,  $\alpha_{A_0}$ , and  $A_0$  as given in Fig.2.

Fig.4: The ratio  $R_\tau$  of the proton lifetime with phases and without phases as a function of  $\theta_\mu$  for the five cases given in Fig.2.

Fig.5: The ratio  $R_\tau$  as a function of the scaling factor  $\lambda$  defined in the text. The four curves correspond to the four sets of input for the parameters  $\tan\beta$ ,  $\xi_1$ ,  $\xi_2$ ,  $\xi_3$ ,  $\theta_\mu$ ,  $\alpha_{A_0}$  and  $A_0$  given by (1)2,  $-1.15, -1.4, 1.27, -1.7, -0.4, 4$  with  $m_0 = 71$  and  $m_{1/2} = 148$  for the point of intersection with  $R_\tau$  axis (dotted)., (2)2,  $-0.87, -1.0, 1.78, -2.15, -0.4, 4$  with  $m_0 = 71$  and  $m_{1/2} = 148$  for the first point (solid). (3)4,  $.5, -1.55, 1.5, 1.56, .6, .8$  with  $m_0 = 550$  and  $m_{1/2} = 88$  for the first point (dashed). (4)4,  $1.5, 1.6, 1.7, -1.56, .6, .8$  with  $m_0 = 750$  and  $m_{1/2} = 88$  for the first point (long dashed). All masses are in GeV and all phases are in radians. All trajectories satisfy edms constraints.

Fig.6: Exhibition of the uncertainties in the proton lifetime predictions due to uncertainties in the input data for case(1) of Fig.2 where we assumed  $M_2/a\Lambda = 0.01$ ,  $M_2 = M_G$ .

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